

PROBLEM SET 2: Simplex Method(Due October 12th)

Important Note: When you submit the Home work, it is necessary to state all definitions and assumptions you make. You will be penalized even for trivial mistakes. So, please turn in your work diligently

$A =$ (last digit of your McGill id)

$B =$ (last but one digit of your McGill id)

For instance, if your McGill ID is 260123457, $A = 7$, $B=5$

$C = 10-A$

$D = 10-B$

$E =$ last digit of $\{100-(\text{Last two digits of your McGill id})\}$

$F =$ last but one digit of $\{100-(\text{Last two digits of your McGill id})\}$

For instance, if your McGill ID is 260123457, $E=3$, $F =4$ ($100-57=43$)

Problem 1

Consider the following LP

Maximize $Z = (16+A)x_1 + (15+B)x_2$

Subject to

$40x_1 + 31x_2 \leq 124+E$

$-x_1 + x_2 \leq 1+F$

$x_1 \leq 3+(C*D)$

$x_1, x_2 \geq 0$

- Solve the problem by the usual simplex method
- Now, instead of choosing to enter the non-basic variable with the highest negative coefficient choose the non-basic variable with the least negative coefficient to enter the basis. Continue this process until you arrive at the optimal solution.
- How do the simplex iterations compare between (a) and (b).

Problem 2

Solve using the simplex method:

Minimize

$Z = (8+A)x_1 + (4+B)x_2 + 6x_3 + 3x_4 + 9x_5$

Subject to

$x_1 + 2x_2 + 3x_3 + 3x_4 \geq 180+C$

$4x_1 + 3x_2 + 2x_3 + x_4 + x_5 \geq 270+D$

$x_1 + 3x_2 + x_4 + 3x_5 \geq 180+E$

and all $x_i \geq 0$

Problem 3

Solve using the simplex method (Do not forget your A's and B's)

(Maximum of 4 iterations)

- For the LP problem formulated from Problem Set 1, Problem 3

- b) For the LP problem formulated from Problem Set 1, Problem 5; use the time matrix provided below

| Swimmer | Backstroke | Breaststroke | Butterfly | Freestyle |
|---------|------------|--------------|-----------|-----------|
| 1 | 65+A | 73+B | 63+C | 57 |
| 2 | 67 | 70 | 65 | 58+E |
| 3 | 68 | 72+A | 69 | 55 |
| 4 | 67 | 75 | 70 | 59 |
| 5 | 71+C | 69 | 75+F | 57 |
| 6 | 69 | 71 | 66 | 59 |

Note: Excel can be used to solve the problems

Problem 4

The following tableau represents a specific simplex iteration. All variables are non-negative.

| | X1 | X2 | X3 | X4 | X5 | X6 | X7 | X8 | Solution |
|----|----|------|----|-----|------|-------|----|----|----------|
| Z | 0 | -5+D | 0 | 4-C | -1+B | -10+A | 0 | 0 | 620+F*A |
| X8 | 0 | 3 | 0 | -2 | -3 | -1 | 5 | 1 | 12-E |
| X3 | 0 | 1 | 1 | 3 | 1 | 0 | 3 | 0 | 6+F |
| X1 | 1 | -1 | 0 | 0 | 6 | 4 | 0 | 0 | 0 |

- Categorize the variables as basic and non-basic, and provide the current values of all the variables.
- Assuming that the problem is maximization, identify all the variables that will improve the value of Z. If each such variable enters the basis, determine the associated leaving variable (if any) and the associated changes in z without using any Gauss-Jordan operations.
- Repeat the above process if the simplex table is an iteration for minimization (remember in this case Z should reduce).
- Which non-basic variable(s) will not cause a change in the value of z, when selected to enter (for maximization or minimization)?

Problem 5

$$\text{Minimize } Z = (2+A)x_1 - (1-E)x_2 + (4+F)x_3$$

Subject to

$$5x_1 + 2x_2 - 3x_3 \geq -(7+F*A)$$

$$2x_1 - 2x_2 + x_3 \leq (8+C*B)$$

$x_1 \geq 0$, x_2 and x_3 are unrestricted.

Set up the simplex problem and complete a maximum of 4 iterations

Problem 6

A company manufactures two products A and B. The unit revenues are $\$(2+A)$ and $\$(3+B)$ respectively. Two raw materials (M1 and M2) used in manufacture of the two products have daily availabilities of $(8+C)$ and $(18+D)$ units respectively. One unit of A uses 2 units of M1 and 2 units of M2, and 1 unit of B uses 3 units of M1 and 6 units of M2.

- Determine the shadow prices of M1 and M2

- b) Suppose that 4 additional units of M1 can be acquired at the cost of 30 cents per unit. Would you recommend the additional purchase?
- c) If M2 availability is increased by 5 units, determine the associated optimum revenue
- d) What is the most the company should pay per unit of M2