

PROBLEM SET 1
(Due September 24th)

Use A, B, C as needed

A = (last but one digit of C)

B = (last digit of C)

C = 100 – (last 2 digits of your id)

Problem 1

An investor has decided to invest a total of \$50,000 among three investment opportunities: savings certificates, bonds and stocks. The annual return on each investment is estimated to be 7%, 9% and 14% respectively. The investor does not intend to invest his annual returns. He would like to maximize his returns while investing a minimum of 10,000 in bonds. Also the investment in stocks should not exceed the combined total investment in bonds and savings certificates. Finally, he should invest between \$5000 and \$15000 in savings certificates. Formulate a Linear program to assist the investor towards making his investment decisions.

Now reconsider the same investment of 50,000 with only two options: bonds and stocks. The annual return on each investment is estimated to be 9% for bonds and 14% for stocks. The investor would like to maximize his returns while investing twice the amount in bonds. Reformulate the LP and use the graphical solution method to obtain the optimal allocation.

Problem 2

A mixture of freeze-dried vegetables is to be composed of beans, corn, broccoli, cabbage and potatoes. The mixture is to contain (by weight) at most 40% beans and at most 32% potatoes, the mixture should contain at least 5 grams of iron, 36 grams of phosphorus, and 28 grams of calcium. The nutrients in each vegetable and the costs are shown below

Vegetable	Milligrams Nutrient per kg of vegetable			Cost per Kg (cents)
	<i>Iron</i>	<i>Phosphorus</i>	<i>Calcium</i>	
Beans	0.5	10	200	20
Corn	0.5	20	280	18
Broccoli	1.2	40	800	32
Cabbage	0.3	30	420	28
Potatoes	0.4	50	360	16

Formulate a Linear Programming model to assist in the development of the mixture.

Problem 3

Larry Edison is the director of the Computer Center for Buckley College. He now needs to schedule the staffing of the center. It is open from 8 A.M. until midnight. Larry has monitored the usage of the center at various times of the day, and determined that the following number of computer consultants are required:

Time of Day	Minimum Number of Consultants Required to Be on Duty
8 AM – Noon	4
Noon - 4 PM	8
4 PM – 8 PM	10
8 PM - Midnight	6

Two types of computer consultants can be hired: full-time and part-time. The full-time consultants work for 8 consecutive hours in any of the following shifts: morning (8 A.M.–4 P.M.), afternoon (noon–8 P.M.), and evening (4 P.M.–midnight). Full-time consultants are paid $\$(14+A)$ per hour. Part-time consultants can be hired to work any of the four shifts listed in the above table. Part-time consultants are paid $\$(12+A)$ per hour. An additional requirement is that during every time period, there must be at least 3 full-time consultants on duty for every parttime consultant on duty. Larry would like to determine how many full-time and how many part-time workers should work each shift to meet the above requirements at the minimum possible cost. Formulate a linear programming model for this problem

Problem 4

A caterer must prepare from five fruit drinks in stock a 500 litres of punch containing at least 20% orange juice, 10% grape juice and 5% cranberry juice. If inventory data are as shown below, formulate an LP to determine how much of each fruit drink should the caterer use to obtain the required composition at minimum total cost?

	Orange Juice %	Grapefruit Juice %	Cranberry Juice %	Supply (liters)	Cost (\$/liter)
Drink A	40	40	0	200	1.50
Drink B	5	10	20	400	0.75
Drink C	100	0	0	100	2.00
Drink D	0	100	0	50	1.75
Drink E	0	0	0	800	0.25

Problem 5

A 400-meter medley relay involves 4 different swimmers, who successively swim 100 meters of the backstroke, breaststroke, butterfly and freestyle. A coach has six very fast swimmers whose expected times in the individual events are given below

Swimmer	Backstroke	Breaststroke	Butterfly	Freestyle
1	65	73	63	57
2	67	70	65	58
3	68	72	69	55
4	67	75	70	59
5	71	69	75	57
6	69	71	66	59

Formulate a LP model to assist the coach in assigning the swimmers to the 400 meter medley race.

Problem 6

Solve the following linear program using the graphical method. Compute the value of the objective function and decision variables at optimality. Also, plot the feasible region.

$$\text{Min } Z = Ax_1 + Bx_2$$

Subject to:

$$4x_1 + 3x_2 \geq 4+A$$

$$6x_1 - 5x_2 = 6+B$$

$$3x_1 + 8x_2 \leq 18+C$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0$$