

Problem 1

Decision Variables:

 $x_{ij}$  where  $i \rightarrow$  factory  
 $j \rightarrow$  retailer (boxes)

 $x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}$ 

Objective Function:

$$\min Z = 14x_{11} + 13x_{12} + 11x_{13} + 13x_{21} + 13x_{22} + 12x_{23}$$

Constraints:

$$\left. \begin{aligned} x_{11} + x_{21} &= 1000 \\ x_{12} + x_{22} &= 700 \\ x_{13} + x_{23} &= 500 \end{aligned} \right\} \text{Demand}$$

$$\left. \begin{aligned} x_{11} + x_{12} + x_{13} &= 1200 \\ x_{21} + x_{22} + x_{23} &= 1000 \end{aligned} \right\} \text{Supply}$$

$$x_{11} \dots x_{23} \geq 0$$

# Problem 2

$$\min C = -2X + Y \rightarrow \max Z = -C = 2X - Y$$

$$\text{subject to: } x + 2y \leq 6$$

$$3x + 2y \leq 12$$

$$x, y \geq 0$$

Augmented Form:

$$\max Z = 2X - Y + \phi U + \phi V$$

$$x + 2y + u = 6 \leftarrow$$

$$3x + 2y + v = 12$$

$$x, y, u, v \geq 0$$

$$\rightarrow Z - 2X + Y - \phi U - \phi V = 0$$

Simplex Table

	$x$	$y$	$u$	$v$	$Z$	$b_i$	min. ratio
$u$	1	2	1	0	0	6	$6/2 = 3$
$\rightarrow v$	3	2	0	1	0	12	$12/3 = 4 \leftarrow$
$Z$	-2	1	0	0	1	0	
$R1 - R2 \cdot u$	0	$4/3$	1	$-1/3$	0	2	
$\frac{2}{3}R2 \cdot X$	1	$2/3$	0	$1/3$	0	4	
$2R2 + R3 \cdot Z$	0	$7/3$	0	$2/3$	1	8	

$\geq 0 \Rightarrow$  optimal Solution!

$$\left. \begin{array}{l} u^* = 2 \quad v^* = 0 \\ x^* = 4 \quad y^* = 0 \end{array} \right\} Z^* = 8$$

$$C = -8$$

### Problem #3

$x_1 \rightarrow$  # townhouses

$x_2 \rightarrow$  # single-storey detached

$x_3 \rightarrow$  # two-storey detached

$$\max Z = 15000x_1 + 18000x_2 + 20000x_3 \quad (\$10000)$$

$$\text{subject to: } 40x_1 + 50x_2 + 60x_3 \leq 2880 \quad (\$10000)$$

$$25x_1 + 30x_2 + 40x_3 \leq 2400 \quad (1000 \text{ hours})$$

$$\frac{1}{6}x_1 + \frac{1}{4}x_2 + \frac{1}{2}x_3 \leq 60 \quad (\text{acres})$$

$$x_1, x_2, x_3 \geq 0$$

Augmented Form:

$$\max Z = 15x_1 + 18x_2 + 20x_3 + 0x_4 + 0x_5 + 0x_6$$

$$\frac{1}{6}x_1 + \frac{1}{4}x_2 + \frac{1}{2}x_3 + x_4 = 60$$

$$40x_1 + 50x_2 + 60x_3 + x_5 = 2880$$

$$25x_1 + 30x_2 + 40x_3 + x_6 = 2400$$

$$x_1, \dots, x_6 \geq 0$$

# Simplex Table

	$x_1$	$x_2$	$x_3$ $\downarrow$	$x_4$	$x_5$	$x_6$	$Z$	$b_i$	min. ratio
$x_4$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{2}$	1	0	0	0	60	120
$\rightarrow x_5$	40	50	<span style="border: 1px solid black; padding: 2px;"><math>60</math></span>	0	1	0	0	2280	48
$x_6$	25	30	40	0	0	1	0	2400	60
$\rightarrow Z$	-15	-18	-20	0	0	0	1	0	<del>0</del>

$R_1 + \frac{1}{6}R_2 \rightarrow x_4$	$-\frac{1}{6}$	$-\frac{1}{6}$	0	1	$-\frac{1}{120}$	0	0	36	-ve
$\frac{1}{60}R_2 \rightarrow x_3$	$\frac{1}{30}$	$\frac{5}{6}$	1	0	$\frac{1}{60}$	0	0	48	<span style="border: 1px solid black; padding: 2px;"><math>72</math></span>
$R_3 - \frac{5}{3}R_2 \rightarrow x_6$	$-\frac{5}{12}$	$-\frac{10}{3}$	0	0	$-\frac{1}{3}$	1	0	480	-ve
$Z$	$-\frac{5}{12}$	$-\frac{4}{3}$	0	0	$\frac{1}{12}$	0	1	960	

$R_1 + \frac{1}{6}R_2 \rightarrow x_4$	0	$\frac{1}{24}$	$\frac{1}{4}$	1	$\frac{1}{240}$	0	0	48	
$\frac{3}{2}R_2 \rightarrow x_1$	1	$\frac{5}{4}$	$\frac{3}{2}$	0	$\frac{1}{48}$	0	0	72	
$R_3 + \frac{5}{3}R_2 \rightarrow x_6$	0	$-\frac{5}{4}$	$\frac{5}{2}$	0	$\frac{5}{8}$	1	0	600	
$R_4 + \frac{5}{3}R_2 \rightarrow Z$	0	$\frac{3}{4}$	$\frac{5}{2}$	0	$\frac{3}{8}$	0	1	1080	

$$x_1^* = 72$$

$$x_2^* = 0$$

$$x_3^* = 0$$

$$x_4^* = 48$$

$$x_5^* = 0$$

$$x_6^* = 600$$

$$Z^* = 1080$$